You Can Count on Your Fingers: The Role of Fingers in Early Mathematical Development

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Abstract

Even though mathematics is considered one of the most abstract domains of human cognition, recent work on embodiment of mathematics has shown that we make sense of mathematical concepts by using insights and skills acquired through bodily activity. Fingers play a significant role in many of these bodily interactions. Paralleling their behavioral importance, fingers are disproportionately represented in the somatosensory and motor cortices in the human brain. The mechanisms that underlie the relation between fingers and number processing have attracted a lot of attention in recent years. Finger-based interactions provide the preliminary access to foundational mathematical constructs, such as one-to-one correspondence and whole-part relations in early development. Furthermore, children across cultures use their fingers to count and do simple arithmetic. In this paper, we synthesize mathematics education and neurocognitive research on the relevance of fingers for early mathematics development. We delve into issues such as how the early multimodal (tactile, motor, visuospatial) experiences with fingers might be the gateway for later numerical skills, how finger sense ability, finger counting habits, and numerical abilities are associated at the behavioral and neural levels, and implications for mathematics education. We argue that, taken together, the two bodies of research can better inform how different finger skills support the development of numerical competencies, and provide a road map for future interdisciplinary research that can yield to development of diagnostic tools and interventions for preschool and primary grade classrooms.

Keywords: cognitive development, numerical cognition, finger counting, finger sense, embodied cognition, neuroscience, mathematics education

Across cultures children and adults use their fingers to count, to communicate about numbers, and to do arithmetic. Body-based counting and arithmetic systems, all involving fingers and sometimes other body parts, emerged independently across cultures, from New Guinea (Saxe, 1981) to ancient Babylon, Egypt, and Romans (Richardson, 1916), and show a wide range of variability (Bender & Beller, 2012). Given the deep relation between fingers and numbers, it is crucial to understand how and why finger counting and other finger-based interactions with numbers relate to numerical development, and how we can develop approaches in mathematics education that can harness the relevance of fingers for numerical development. In addition, the finger and mathematics relation constitutes an important aspect of embodiment of mathematics (Domahs, Moeller, Huber, Willmes, & Nuerk, 2010; Lakoff & Nunez, 2000); how bodily systems ground and scaffold mathematical development. Understanding how the finger sensorimotor system grounds numerical processes can also inform the wider question of how bodily systems, support and scaffold cognitive skills.

The foundations for mathematical abilities, like all cognition, can be traced to early development and have neurological bases that are linked to the active experiences of children. Children's bodies, particularly hands and fingers, play a crucial role in grounding and in establishing the neural networks that underlie numerical abilities (Butterworth, 1999; Moeller et al., 2012). The bodily activities that we engage in have a direct impact on brain development and future cognitive processing (Greenough, Black, & Wallace, 1987; Posner & Rothbart, 2007). This is particularly true of children due to the rapid neural development that occurs in early life. Therefore, choosing activities that will provide foundational experiences to ground and support the development of more advanced cognitive skills is very important. In the case of fingers, there is no consensus on what form these activities should take or how fingers and hands play a role in numerical development. For example, a recent article in *The Atlantic,* titled "Why kids should use their fingers in math class," reports on a tendency in schools towards discouraging finger use in math classrooms, despite research that shows an intricate relation between fingers and early numerical skills, and mathematics educators advocating for use of finger-based strategies early on (Boaler & Chen, 2016). As Cowan (2013) noted, "Although an earlier generation considered all methods apart from retrieval as crutches and teachers were likely to forbid the use of fingers, we [mathematics educators] now see these as a necessary part of children's development" (p. 56).

Our goal in this paper is to synthesize findings in mathematics education, and cognitive science/neuroscience (neurocognitive) research, on the relevance of fingers for numerical development, and reflect on gaps in our knowledge, in an effort to lay out a road map for future research and practice. Mathematics education and neurocognitive research on the relevance of fingers for mathematical cognition differ in terms of theoretical orientations, goals and methodologies used. Moeller et al. (2011) have previously argued that neurocognition and mathematics education present opposing views on effects of finger counting on numerical development. According to their characterization, while "mathematics education research recommends the reliance on external representations, including finger-based ones, only as an aid in the transition to mental representations of numbers" (p.4), neurocognitive evidence points to a deep and sustained role of embodied representations, including finger-based ones, in numerical cognition. Our synthesis here presents a slightly different picture. We argue that constructivist approaches in mathematics education, and neurocognitive research, from an embodied cognition theoretical framing, can be complementary in understanding the finger and mathematics relation, in spite of theoretical and methodological differences.

Mathematics education research presented here follows a constructivist orientation and explores ways with which bodily interactions, including fingers, contribute to the active construction of number concepts. Methodologically, these studies follow traditions of genetic epistemology (Piaget, 1970) and present support for theoretical claims based on clinical interviews and classroom observations. Neurocognitive research focuses primarily on the cognitive and neural mechanisms that support number processing, and the developmental and evolutionary origins of these mechanisms. The theoretical claims, under this body of research, are supported by behavioral and neuroimaging (e.g., fMRI, EEG/ERP) data from lab experiments and intervention studies. Taken together, these two bodies of research can provide new insights into the relation between finger and number processing, and the forms of bodily interactions that can support numerical development during development.In the first part of the paper we review research that connects development of early perceptual abilities with understanding whole-part relations and learning how to count. It is argued that the concept of "number" is grounded in our ability, first to perceive distinct objects, then to categorize and group them based on a measure of "sameness" (Glasersfeld, 1981; Steffe, Cobb, & Glasersfeld, 1988). Our abilities to recognize distinct objects based on their perceptual features, and to categorize them in groups of plural objects relies on our bodily interactions with the world. From this perspective, numbers are related to how we perceive and interact with the world, and our readiness to understand numbers very much relates to our early bodily interactions.

After covering how bodily interactions in early development contribute to the development of *number sense,* we explore how finger counting strategies emerge and evolve, and the relevance of finger counting to mathematics learning difficulties. Then we focus on neurocognitive studies on the relation between finger counting, finger sense and mathematics learning and development. Finally, we discuss ways of bridging neurocognitive and mathematics education perspectives, and explore crucial questions for future interdisciplinary efforts.

Different Approaches to the Role of Fingers in Early Numerical Development

There are several perspectives that have been taken over the years in regards to fingers and mathematics. According to one constructivist perspective fingers provide a physical and accessible representation for ordinal and cardinal representations in early development, and finger counting strategies facilitate arithmetic learning. These strategies evolve as a result of practice, automatization, and development of composite unit understanding (e.g., from count-all to count-in strategies, to count-on), and are gradually replaced by computational strategies supported by verbal, symbolic, and visuospatial representations (Baroody, 1987; Dehaene & Cohen, 1995; Steffe et al., 1988). This *transition model* paints a picture of early numerical development characterized by representational transitions from concrete to more abstract forms of processing, and provides us with a description of how the concept of numerosity emerges as a result of interacting with the physical world. An alternative approach, which is partially complementary with the constructivist perspective, interprets the finger and number relation from an embodied cognition perspective (e.g., Domahs et al., 2010; Moeller et al., 2012). Here we review and compare these approaches, and discuss ways with which these two main approaches can together present a more comprehensive account for the role of fingers in development of numerical skills.

Constructivist Approaches: From Sensorimotor to Figural Unit Items

Built on Piaget's work (1954) on how children develop an understanding of whole-part relations, Steffe, Glasersfeld and colleagues (1988; 1983; 1985) traced children's ability to count to early perceptual experiences. Before counting and understanding numerosity, infants develop the ability to perceive discrete objects –*perceptual unit items*– by abstracting aspects of their ongoing sensorimotor experience. Steffe et al. (1988) proposed that "… isolating something from the experiential continuum is an indispensable prerequisite for any conception of number" (p.3). Glasersfeld (1981) pointed out that perceiving an entity as an "object" requires coordination of sensory material from multiple sources. Most often, this would be a combination of visual and tactile information. Glasersfeld proposed a pulse like theory of attention, where the perception of a sensorimotor unit (an object) is bounded by states of unfocused attention, and includes pulses of attention in between states of unfocused attention. Each pulse of attention focuses on a different aspect of the sensorimotor experience (e.g., in perceiving a cup these might include features of color, texture, shape). There are two prerequisites for a sensorimotor item to be recognizable: First, it needs to contain more than one focused impulse, which "make the thing qualitatively discriminable and give it location in the experiential field" (p.88). The second aspect is "boundedness." The sensory-motor item needs to be distinguishable from its background. The pre-verbal ability to perceive distinct objects is followed by the ability to label objects. By about 2 years of age children most children can not only perceive objects but label them, and use the plural form of the labels. The use of plural requires not only extraction of certain aspects of the fuzzy sensorimotor experience as a distinct "thing", but also recognizing the recurrent nature of this unit across different instantiations.

Steffe et al. (1988) distinguished between pluralities and collections. Pluraties refer to more than one instantiation of a previously extracted sensorimotor experience; however, it is not bounded in any specific way. The plural refers to more than one, but not to a collection, because what it refers to has no beginning or end. Collections are bounded, and they refer to pluralities defined with a beginning and end. For example "the cups on the table" is a collection; the items in the collection can be considered as recurrences of a perceived unit due to similarities in the sensorimotor experiences they elicit. They are bounded by the perceptual field presented by the table, and therefore they are countable.

Through development, the act of counting becomes progressively more independent from immediate perception. In addition to perceptual unit items, Steffe et al. (1988) proposed four counting types to characterize the gradual independence of counting from immediate sensorimotor interaction. Figural unit items are visualized substitutes for objects that are not visible (e.g., when some units in a collection are partially blocked, children assume that they are there by imagining or visualizing them as part of the collection). Motor unit items are motor movements (e.g., pointing with the index finger) that substitute for the perceptual units. Verbal unit items are utterances of a number word, without the need to present the physical perceptual unit item. And finally, abstract unit items represent complete independence from the sensorimotor material in the counting task. For example, when the child realizes that the utterance of the number word "eight" both represents the number word sequence "one, two, …, eight" and a collection of eight discrete items, the child is said to have "an abstract conception of number" (p.6). For example, when asked how many checkers were hidden, in a situation where the total is 12 and eight is visible, if the child counts "9 as 1, 10 as 2, 11 as 3, 12 as 4", then she is referring to each verbal item as a countable unit. At this stage the abstract, verbal item replaces the sensorimotor item.

When we consider infants' pre-verbal, bodily experiences, finger-based interactions standout as, perhaps, the most significant of all bodily experiences, given that the movement of and tactile sensations with (both of which are related) fingers become progressively more distinct. Infants spend time looking at their hands and fingers and watch their fingers move in progressively more independent ways. The sensorimotor system for fingers involve tactile, motor and visual modalities, and early physical experiences serve for the integration of these modalities to develop visually guided fine motor movements and tactile perceptual abilities (Barrett, Traupman, & Needham, 2008). It could be said that fingers are discrete sensorimotor units that are attached to our body. If extracting "distinct things" from our sensorimotor experience is a prerequisite to understanding plurality and collections, and eventually to understanding numerosity (Steffe et al., 1988, 1983), then experiences with fingers should be the gateway for these competencies. Fingers are distinctly presented across multiple modalities (i.e., tactile, modal, visual), they are attached to our body as two separate collections of five units, and they can be moved independently to create different configurations (e.g., the simplest way being an open or closed state, which is useful in counting and representing collections of different number of units).

Even before fingers are used for explicitly representing numbers, they provide the preliminary and grounding sensorimotor experiences for perception of discrete units. During finger counting, fingers act as motor unit items. Shortcut finger counting strategies, like counting-on, represent aspects of abstract unit items, and are likely to pave the way for the symbolic representation of numbers. For example, while calculating "8 + 5" the counting-on strategy (starting to finger count from nine up to 13, instead of first counting from one to eight) includes three different counting types (i.e., figural, verbal, and abstract) (Steffe & Glasersfeld, 1985).

Even though Steffe et al. framed the development of the number concept from a constructivist standpoint, their approach is different from Piaget's original formulation of how sensorimotor experiences shape numerical development. Glasersfeld (1981) pointed out that even though Piaget had referred to children's understanding of whole-part relations, he did not provide an account for how this understanding develops: "In all his work on the development of number, Piaget focuses on a conceptual complex that involves class inclusion and order, and, like most of his predecessors, he takes the construction of units for granted" (p.84). Steffe, Glasersfeld and colleagues (Steffe et al., 1988, 1983; Steffe & Glasersfeld, 1985) filled this gap by explicating how perception of sensorimotor units eventually lead to abstract representations for numbers, through a series of representational shifts, including motor, figural, and verbal unit items.

Embodied Approaches

Alternatively, the finger and mathematics relation can be considered from an embodied cognition perspective. Embodied cognition is not a single and unified theory of cognition, but rather a transdisciplinary research program with a multitude of claims and theories, with the shared notion that cognition is grounded in bodily systems (Clark, 1999; Gallese & Lakoff, 2005; Smith & Gasser, 2005). According to the embodied account of mathematics, bodily interactions do not aid development of abstract, and eventually disembodied, representations and logical principles, but rather help structure the sensorimotor systems in a way to provide semantic content for number processing (de Freitas & Sinclair, 2013; Gallese & Lakoff, 2005; Nunez, 2012; Sfard, 1994). The use of symbolic representations (e.g., Arabic numerals, algebraic notations) does not mean that cognitive processing becomes progressively more disembodied, and takes a purely abstract and symbolic processing form. Instead, symbolic processing itself is grounded in the sensorimotor systems. For example, Barsalou (1999) argued that during perceptual experiences, association areas in the brain capture bottom-up sensorimotor patterns. Later, during the use of perceptual symbols, association areas activate some of the same sensorimotor areas in a top-down manner; meaning that certain perceptual experiences can trigger associated sensorimotor states. From this perspective, the meaning of symbols (semantics) emerge from the sensorimotor simulation of relevant systems. The early finger-based interactions and finger counting experiences not only aid numerical development during childhood, but also ground and shape the number processing system, extending its effects to how adults process numbers (see Berteletti & Booth, 2016 for a review). From the embodied cognition perspective, number processing and bodily systems do not gradually become independent. According to the embodiment thesis (Lakoff & Nunez, 2000; Soylu, 2011) the number processing system continues to be embodied through development and adulthood, and is grounded in sensorimotor systems. In other words, the sensorimotor system is part of the number processing network, instead of just being a precursor to or constituting the foundation for it.

Even though Steffe et al. (1988) have not extensively theorized about what role the sensorimotor system plays once abstract representations (abstract unit items) are constructed, they hinted at some form of sensorimotor simulation: "The transition from being a counter of perceptual unit items to a counter of verbal unit items involves the internalization of sensory-motor activity" (p.6). Separately, in their explanation of what they referred to as the "uniting" operation (distinguishing discrete units from collections) Steffe and Glasersfeld (1985) added, "A mental operation is an interiorized action, an action that can be carried out in thought. What may be overlooked is that these mental acts lead to a result and must have material to operate on. The uniting operation can have a collection as material and a unit of units - a whole number - as a result." (p. 272). Nevertheless, they fell short of describing what is meant by "internalization of sensory-motor activity" or the nature of an "interiorized action." Embodied accounts of mathematics, especially sensorimotor simulation theories (Soylu, 2011, 2016; Svensson, 2007), address this missing component. The embodied account is corroborated by neurocognitive studies showing that the finger sensorimotor network continues to play a role in adults' number processing (Andres, Davare, Pesenti, Olivier, & Seron, 2004; Badets, Andres, Di Luca, & Pesenti, 2007; Di Luca & Pesenti, 2008; Rusconi, Walsh, & Butterworth, 2005; Soylu & Newman, 2016), and that early finger counting experiences leave a lasting effect on adults' performance and neural correlates for number processing (Newman & Soylu, 2014; Tschentscher, Hauk, Fischer, & Pulvermuller, 2012). The grounded model argues that number processing does not become disembodied with age and instead early bodily interactions help establish the number network and leave a lasting trace on numerical cognition.

Finger Counting and Finger Sense

For the purposes of this article, we distinguish between two forms of finger processing: finger sense and finger counting. Finger sense (also called finger-localization; Benton, 1955; and finger gnosia; Noel, 2005) is the ability to localize the stimulation of fingers and is, in part, a measure of the preciseness of discriminating regions of sensory stimulation. Various tests have been used to measure finger sense (Benton, 1955; Fayol, Barrouillet, & Marinthe, 1998; Noel, 2005); all involving stimulation of one or multiple fingers, through the touch of a physical object, while the hands are not visible (either because the eyes are closed, or the hands are covered), and the participants being asked to report the fingers touched either verbally, or by moving the matching fingers on the other hand. There are a multitude of studies, which we will cover in detail, that showed that finger sense scores correlate with or predict mathematical skills (e.g., Fayol et al., 1998; Newman, 2016; Noel, 2005).

Finger Counting

Finger counting is universal and ubiquitous across cultures, and show a high-level of cultural variability (Bender & Beller, 2012; Butterworth, 1999). In some cultures body-based counting systems are used, which include other body parts like feet, arms, the head, in addition to fingers, and in others, higher-order counting configurations are used to represent numbers bigger than 10, for example by using one hand to count numbers from one to five, and using the other one to keep track of how many times counting on the other hand was completed (see Bender & Beller, 2012 for a review of the wide range of finger counting systems used across cultures). The most prevalent modern form of finger counting involves a one-to-one correspondence with numbers from one to 10, with variations on some aspects of it, for example which hand one starts to count with, or which finger is first used on each hand (e.g., thumb or index). One notable exception to this is the Chinese finger counting system, which uses fingers only on one hand, and uses symbolic gestures for number six to 10 (no one-to-one match with fingers); some of which resemble the written Chinese numeral characters (Domahs et al., 2010).

Compared to cultural differences, we know very little of how sociocultural factors, like socioeconomic status, affect early finger-based interactions, and finger counting habits and skills. Previously it was shown that kindergarteners from low-income households use their fingers less often to count and add than children from middle-income households (Jordan, Huttenlocher, & Levine, 1992; Jordan, Kaplan, Ramineni, & Locuniak, 2008). Finger use, as well as finger sense, have been shown to positively predict mathematical achievement in children (Fayol et al., 1998; Noel, 2005; Penner-Wilger & Anderson, 2013). In addition, several studies (Crollen & Noel, 2015; Fuson, 1988; Soylu & Newman, 2016) suggested that finger processing may play a role in setting up the neural networks on which more advanced mathematical computations are built.

If the relation between fingers and numbers goes beyond use of finger counting strategies between ages four to eight, and if the finger sensorimotor system grounds and scaffolds the development of numerical skills, then we need to reevaluate approaches to finger counting and finger skills (particularly finger sense) in mathematics education.

When children first start learning to solve arithmetic problems, they use their knowledge of counting, which is often executed with the help of fingers. The use of finger counting strategies eventually results with memorization of basic arithmetic facts, which then leads to a shift from finger counting strategies to memory-based strategies (Jordan et al., 2008). The transition from finger (and verbal) counting strategies to automatic fact retrieval allows students to focus the limited processing resources to more complex aspects of arithmetic like multi-digit subtraction/addition, long division or complex multiplication (Gersten & Chard, 1999).

Children progressively develop more efficient finger counting strategies during early arithmetic learning. According to Baroody (1987), *counting-all* is the most basic strategy, which involves counting fingers out one by one to represent the first, then the second addend, consecutively until all fingers put out are counted to determine the sum (e.g., for "3 + 2", thumb, index, and middle fingers are opened, then ring and little fingers; all of them are counted together to find the sum). If the addends are both smaller than five, then each addend can be represented separately on each hand, which may not require counting each addend if the finger configuration for numbers from one to five is automatized, and the child has to count only once. A more developed strategy is to bypass the sum count by automatically recognizing the sum either visually (i.e., subitizing) or kinesthetically. A more efficient alternative to the counting-all strategy is the *counting-on* strategy, where the child counts not from one but starts with the cardinal designation of the first number. This strategy can be made even more efficient by always starting with the larger number, which reduces the counting length (e.g., 8 + 3 instead of 3 + 8). Children's use of strategies that imply knowledge of commutativity (e.g., always starting with the larger number) does not warrant knowledge of commutativity (Baroody, 1987). For example, a child who prefers to start counting from the larger number in an arithmetic task might still claim that 3 + 5 is not the same as 5 + 3.

Finger counting and mathematics learning difficulties. Children with mathematics learning difficulties show differences in reliance on and habits of finger counting. First, second, and third grade children with mathematics difficulties are more reliant on finger counting strategies, and they have a harder time transitioning from finger counting to verbal counting and retrieval strategies, which typically occurs towards the end of first grade and early second grade with children who are not identified with any impairments (Bryant, 2009; Geary, 2004; Jordan & Hanich, 2000; Jordan et al., 2008). In addition, mathematics learning disability seems to interact with reading disability; children with both types of disabilities (comorbid) show further problems in memorizing and retrieving arithmetic facts than children with only mathematics learning disability. At first grade children with MD/RD (mathematics and reading disability together) or MD (mathematics disability only) show more counting-procedure and retrieval errors, and use more efficient finger counting strategies less frequently compared to children who are not identified with any impairments (Gersten & Chard, 1999). At second grade, children with MD develop more efficient finger counting strategies compared to their peers with MD/RD (Gersten & Chard, 1999; Jordan & Montani, 2015).

The fact that children with MD/RD show lower accuracies with both finger and verbal counting strategies compared to the group with MD-only was proposed to hint at a weakness with counting procedures for children with MD/RD and with mental computation (e.g., fact retrieval) for children with MD-only (Jordan, Hanich, & Kaplan, 2003). There is evidence showing that, similar to dyslexia (which results from a deficit in the phonological system; see

Schlaggar & McCandliss, 2007), mathematics disability is not due to general low intellectual performance, but is more likely to be due to a more specific deficit that affects number processing and other closely related processes (see Rousselle & Noel, 2007). There is some controversy, however, as to which aspect of the number processing system is affected in mathematics disability (Devine, 2013; Rubinsten & Henik, 2009). There is no single cause that explains all cases of MD, therefore it is likely that MD represents a spectrum of conditions involving one or more deficits in various number-related processes.

Unimpaired children transition from finger counting strategies to arithmetic fact retrieval strategies from first grade to second grade, while children with math-learning and math-learning/reading disabilities do not show this shift and continue to rely on finger counting even after first grade (Gersten & Chard, 1999). Since fingers can function as a working-memory aid in keeping track of addends, difficulties with representing numerical magnitudes in the working memory during addition is one of the explanations proposed for why children with MD rely on finger counting for a more extended amount of time (Geary, 2004). Working memory deficiency is also proposed also explain why children with MD undercount or overcount during finger counting, since miscounting might be due to losing track of what has already been counted and what remains to be counted (Geary, 1990; Hanich, Jordan, Kaplan, & Dick, 2001). Given the further severity of mathematics disability of children with both MD and RD, the working memory deficiency might also be related to language-based information representation. If phonetic representation of number words cannot be retained in working memory or misrepresented due to a problem in the phonetic-articulatory system, children might resort to relying on finger representations, which bypass the phonetic representation (Geary, 1990).

Finger counting and number skills. One way to approach the finger and numerical development is that fingers provide a "natural scaffold for calculation" (Jordan et al., 2008, p. 662). As with all manipulatives, however, there is controversy as to whether finger use during problem-solving is helpful or detrimental. This may depend on the timing of finger use. Evidence suggests that finger use *early* is helpful. For example, Jordan and colleagues (1992; 1994) found that finger use was linked to higher accuracy on number combinations (includes responding to questions like 'How much is 9 take away 2?') in kindergarten and first grade students. Those students who rarely used finger counting spontaneously had poorer performance. By second grade, however, there was a shift in that better mathematical performance was associated with reduced finger use and a greater reliance on retrieval strategies (Jordan et al., 2008). These findings may reflect a developmental trajectory in which finger counting sets the stage for more advanced skills, but once those skills are acquired finger counting is no longer needed; possibly because finger counting is supposed to become, what Steffe and Glasersfeld (1985) referred to as, an "interiorized action" (or alternatively an embodied simulation) by then. It follows then that children who are still finger counting are the children who have not acquired those more advanced grade level skills. By analogy, a kindergartener who uses invented spelling (writing "kitten" as KTTN) shows signs of precocity and readiness to learn to read; however, the same behavior in a 4th grader is a negative indicator of age-appropriate literacy (Treiman & Zukowski, 1991).

Steffe and colleagues' (1988, 1983) interpretations of their own findings on numerical development in elementary children may be helpful in understanding the developmental question. They proposed a stage theory of counting suggesting that children's learning of number concepts follows levels of increasing sophistication according to their ability to conceptualize and interiorize individual units. Fingers (which are used to count all kinds of different things) stand in between the concrete things that are counted and the abstract concept of number itself, and therefore help children develop an initial level of numerical representation that is more abstract than touchable items and through which addition and subtraction may be understood and generalized. As children become more competent in their numerical skills, however, they will discard the use of fingers or other physical representations, as the conceptualization of numbers becomes more abstract. In this regard, finger counting and other finger activity can be seen as an indicator of the students' level of numerical development, and preference for using a tool that provides increased numerical power, until they advance to the point where the use of fingers is no longer necessary.

What is it about finger counting that could be so important? Steffe and colleagues' theory about fingers constituting a level of abstraction between specific things and abstract concepts of number is one possibility. There is growing evidence indicating that fingers play a significant role in the development of a mature counting system, inasmuch as early counting consists of touching in sequence the thing counted, as one says the number name (Butterworth, 1999, 2005, Fuson, 1982, 1988). Others have suggested that fingers serve as a memory aid during counting (Geary & Wiley, 1991), and that fingers aid in understanding cardinality (Fayol & Seron, 2005) and the development of the one-to-one correspondence principle (Alibali & DiRusso, 1999). Indeed, Rusconi et al. (2005) suggested that "counting and combining quantities on one's own fingers seem to represent an obligatory passage to the mastery of number concepts and of arithmetical operations" (p. 1610).

Starting at a very young age, even before number symbols are learned, a link is created between magnitude and fingers, with children being able to represent numbers with their fingers as early as three years of age (Fuson, 1988). This may simply be because magnitude is first mapped onto a body-based system and then translated into a more abstract numeral system, where numbers and fingers are connected. This is essentially the hypothesis proposed by Butterworth (1999b), who suggested that fingers and numbers link because fingers are used to represent numerosity. If this is the case, then individual experiences in finger counting should matter. There is evidence from studies on the relation between finger counting habits and neural correlates of number processing that is consistent with this proposal. Tschentscher et al. (2012) presented number words to adults and found that the fMRI-measured brain activation observed was systematically related to finger counting habits: those who began counting with the left hand, "left-starters," showed increased activation in the right motor cortex, while "right-starters" showed more activation in the left motor cortex (no motor response was required during the task). Because finger counting habits have been found to be stable across development (Sato & Lalain, 2008), this finding suggests that while adults may not use their fingers in the service of mathematical calculation very often, those early experiences relating fingers with number (and magnitude) create a lasting neural impression that was activated on seeing a number word, even when no motor response was required.

Finger Sense

Thus far we have focused primarily on the role of finger counting in numerical and mathematical development. However, it may be that finger *sense* is the building block upon which finger counting and numerical competence rests. There is a growing body of research focused on the association between finger sense and numerical and mathematical competency. For example, in a study examining whether finger sense was foundational to number representation, Noel (2005) examined first graders and found that finger sense in first grade was correlated with the ability to map between numerals and their associated magnitudes in second grade.

The relationship between finger sense and finger counting has not been well studied. Finger sense has been found to be correlated with number knowledge (Newman, 2016; Noel, 2005; Penner-Wilger & Anderson, 2013), which in turn is essential to mathematical performance, and finger counting appears to be an important and possibly necessary part of early mathematical calculation skill development (Moeller et al., 2012). But how finger sense and finger counting relate to each other is not clearly understood. One possibility is that finger counting depends on finger sense; alternatively finger sense itself may be tuned by activities such as counting. Reeve and Humberstone (2011) demonstrated that finger sense ability and use of finger counting for arithmetic co-develop between ages 5 and 7, and that together, but to a lesser extent, with visuo-spatial working memory, finger sense ability predicted both use of finger counting strategies for arithmetic and calculation ability (i.e., high finger sense ability group used more finger counting and had higher calculation ability). While this result does not weight in towards either of the casual directions suggested, it does show that finger sense, finger counting, and calculation abilities co-develop. The co-development of finger sense, counting, and arithmetic skills might explain some of the conflicting findings on the relation between finger sense and arithmetic skills in 5 to 8 year old children; some reporting strong associations (Fayol et al., 1998; Noel, 2005), while others showing no behavioral associations (Long et al., 2016; Newman, 2016; Soylu, Raymond, Gutierrez & Newman, 2017) for roughly the same age groups. Newman (2016) reported that 5 to 8 year old children failed to show a relationship between addition performance and finger sense, while children 9 to 12 year old did show such a relationship; finger sense better predicted performance in older children than younger children. The explanation provided for the discrepancy, between the two age groups on how finger sense relates to addition performance, was that both addition skills and finger sense are still developing in the younger group. However, both skills should be developed within the older group (9-12 yo), making the relationship between these two factors more evident. In a recent fMRI study with 7 and 8 year old children Soylu et al. (2017) reported that even when no behavioral correlations between finger sense and arithmetic (single-digit addition and subtraction) performance were found, activations in particular visuospatial brain areas related to number processing (i.e., left fusiform and lingual gyri, and bilateral precuneus) were found to negatively correlate with finger sense scores –high finger sense children showing reduced activations in these areas during both addition and subtraction. They also found a negative correlation between finger sense scores and activation in the left inferior parietal lobule only during addition, but not during subtraction. They proposed that arithmetic fact retrieval is related to finger sense at the neural level, both for addition and subtraction, even when behavioral correlations are not observed. Given the differential correlation in the left IPL for addition, the nature of this relation was proposed to be different for addition compared to subtraction, possibly due to differences in finger-counting strategies used for these two operations.

Improvements in finger sense may positively impact the use of finger counting in young children via two mechanisms. The first possible route is via motoric processing. For example, speculatively, better finger sense allows for better fine motor skills, which may be necessary for both finger counting and for counting small entities (like rows of counters). Finger sense exercises, and/or activities involving counting on fingers and objects may foster an increased ability to individuate the fingers, which in turn may lead to better finger counting. A second possibility is that finger sense has a direct impact on number sense, perhaps through the discrimination of numerical quantities (Halberda & Feigenson, 2008; Mazzocco & Thompson, 2005), which in turn leads to better finger counting.

Critically, the construct of finger sense is not exclusively or originally about number concepts. The ability to localize the stimulation of fingers (finger sense) is, in part, a measure of the preciseness of discriminating regions of sensory stimulation (but as we will discuss, it also involves more than that). Poor finger sense has been used as an indicator of brain dysfunction and learning disability for several decades (Benton, 1979; Critchleey, 1953; Gerstmann, 1940). More, current research shows that finger sense predicts numerical performance in children (Newman, 2016; Noel, 2005). To understand how finger sense may relate to number and mathematics, it is useful to explore a process model of the typical finger sense test.

During a finger sense test, Noel (2005) had participants sit with both hands palm down on a table in front of them with their hands obscured so that they could not see them. The experimenter, with a pointer, touched the fingers (either just one or a combination of two) in a pre-determined order. After each touch (or combination of touches) the participant was expected to indicate which finger(s) was touched either by moving the corresponding finger of the other hand or pointing to the finger that was touched on a displayed picture. In order to correctly respond, not only did the participants had to "sense" the touch, but they also had to generate an internal representation of their hands and fingers. This internal representation is an internal information model (either an image or propositional representation) of the hand and fingers that contains spatial information regarding the location of the individual fingers. In order to respond correctly during the finger sense task participants had to map this internal representation of their hand and fingers onto another representation, either their opposite hand or a picture of a hand. This mapping process is the determination of a one-to-one correspondence of the representation of the hand being touched and the representation through which the participant indicates their responses.

Thus, finger sense tasks are measuring more than just the ability to discriminate the finger touched, they also measure the ability to activate an internal representation and then map it onto another representation, which requires a host of processes, required for response generation, including working memory and spatial processes –this might decrease reliability and could be considered a weakness of the task–. The ability to generate internal representations of physical objects is, in general, an important cognitive process. But fingers, as part of the body, are special. There is a direct mapping of fingers in somatosensory and motor cortices (e.g., touching the thumb activates a particular part of somatosensory cortex). Learning to generate precise and finely tuned representations of one's own body and to map the spatial relationships of body parts may scaffold learning about external objects. Some support for this idea can be seen in the study of children with motor processing deficits. In a study by O'Brien et al. (2002), children with developmental dyspraxia, an inability to choose, plan, sequence and execute movements, were found to show deficits in global spatial processing, which may implicate a link between finger sense (as defined by the task) and spatial processing in general. In fact, Newman (2016) found a strong relationship between finger sense and matrix reasoning (a non-verbal reasoning test, which involves a series of figures representing a pattern with one figure left blank) scores; children (5-12 years of age) with high finger sense also having higher matrix reasoning scores. The negative correlations found between finger sense scores and activations in three visuospatial processing areas (i.e., left fusiform, and bilateral precuneus) during addition and subtraction, reported by Soylu et al. (2017), provide further neural evidence for the partially visuospatial nature of the link between finger sense and number processing.

The finger sense task can also be considered one of perceptual discrimination (i.e., requires distinguishing between sets of stimuli based on perceptual features). Recent research on the relation between perceptual discrimination, in this case visual, and mathematical skills suggests another potentially related pathway. In particular, the ability to distinguish small differences in the set sizes of arrays of objects (a set of 100 stars versus a set of 108 stars) as a preschooler has been shown to predict later success in mathematics (Halberda & Feigenson, 2008; Mazzocco, Feigenson, & Halberda, 2011). Discrimination of discrete quantity is trainable in adults, although it is not clear how well training transfers to mathematical problem solving (DeWind & Brannon, 2012), a limitation that might be due to the fact the participants were adults (which might make it harder to modulate the established problem solving system). In a study of children from low-income families (ranging in age from 44 to 71 months), Fuhs and McNeil (2013) found that the approximate number system (ANS) acuity (i.e., the ability to correctly compare which of the two collection of dots represent a larger numerosity based on an approximate judgment, without counting) did not predict mathematics ability (including measures like counting skills, number facts, calculations skills) when controlled for inhibitory control. This result is in contrast to previous studies where ANS acuity was found to correlate with mathematics ability in comparable age groups (Libertus, Feigenson, & Halberda, 2011; Mazzocco et al., 2011). Fuhs and McNeil (2013) explained this discrepancy based on lack of opportunities for low-income children to connect their ANS with symbolic mathematics skills. A separate body of research (e.g., Jordan, Kaplan, Ola, & Locuniak, 2006; Ramani & Siegler, 2008) showing that children from low-income homes have less exposure to early number concepts supports this argument. Even though multiple studies have shown that finger sense ability predicts mathematics ability in children (Newman, 2016; Noel, 2005; Penner-Wilger & Anderson, 2013; Wasner, Nuerk, Martignon, Roesch, & Moeller, 2016), no previous study has explored if the finger sense and mathematics ability relation holds true for children from low-income homes. It is possible that early association of finger sense ability with number skills depends on exposure to number concepts in the home environment before formal education.

The Relation between Finger and Number Representations in the Brain

The different theories that explain what underlies the finger and number relation can generally be categorized into three as, localizationist, developmental (functionalist) and evolutionary (neural reuse) approaches (Penner-Wilger & Anderson, 2013). According to the localizationist account (Dehaene, Piazza, Pinel, & Cohen, 2003) co-occurrence of symptoms related to finger and number processing in neuropsychological cases (e.g., lesions in left angular gyrus; Gerstmann, 1940) and evidence for neural overlap between number and finger processing are due to high anatomical proximity of crucial neural resources for finger and number processing, and not due to a functional relationship between the finger and number processing systems.

According to the evolutionary account, the finger sensorimotor system is involved in number processing because of a general evolutionary phenomenon named "neural reuse" (Anderson, 2010). Given that the human brain has not gone through a significant evolutionary change to accommodate new cognitive skills, like verbal language and mathematics, the neural reuse theory asserts that new cognitive skills rely on and reuse existing bodily systems, which originally evolved to realize other, more bodily functions (e.g., for fingers, visually guided fine motor movements). In the case of number processing, the finger sensorimotor circuits, which allow independent multimodal representation of fingers, are reused to represent and process numerical quantities (Penner-Wilger & Anderson, 2013). The evolutionary account is not necessarily mutually exclusive with the developmental account. While the finger and mathematics relation might have some genetic bases, the development and fine tuning of the number processing network might still heavily rely on bodily experiences during development.

The number processing network in the brain is distributed across many areas. According to the Triple Code Model (TCM) –a model that has significantly influenced discussions on the neural correlates of number processing in the last two decades- three parietal areas, each matching a distinct form of numerical representation, constitute the core neural correlates for number processing (Dehaene & Cohen 1997; Dehaene et al., 2003). First, a non-verbal magnitude representation system, located bilaterally in the intraparietal sulcus (IPS), is involved in processing of numerical magnitudes (e.g., size and distance relations between numbers). Second, a visual number representation system, supports recognizing and encoding Arabic numerals, and is located in bilateral posterior superior parietal areas. Finally, a third verbal representation system, mainly located in the left angular gyrus (AG), is involved in processing verbal number information (e.g., retrieving arithmetic facts from phonological memory). In a later study, left inferior frontal gyrus (IFG) and bilateral supplementary motor areas, were proposed to be included in TCM for their supportive role for magnitude processing and application of rule-based heuristics in mathematical operations (e.g., as in arithmetic), respectively (Klein et al., 2014).

While these parietal areas are crucial for number processing, overlapping areas in the parietal cortex were also found to be related to fingers. The study of the overlap between number and finger processing in the parietal cortex goes back almost a century. In 1924 Josef Gerstmann diagnosed an adult patient who was not able to name her own fingers or point to them on request. Tests on this patient also revealed that she had difficulty differentiating between her right and left hand, or another person's right and left hands. In addition, she performed poorly on calculation tests and had impairments in spontaneous writing. The source of the symptoms was a lesion located in the left AG (Gerstmann, 1940). It was these studies by Gerstmann in the 1920s that first linked fingers to number processing at the neural level. Since then, a number of studies have replicated Gerstmann's findings, with patients with parietal lesions (Mayer et al., 1999). In addition, two magnetic stimulation studies with healthy adults parallel findings from patient studies, showing that stimulation of angular gyrus leads to disruptions both in number processing and finger sense (Roux, Boetto, Sacko, Chollet, & Tremoulet, 2003; Rusconi et al., 2005).

Gerstmann's syndrome is controversial and the existence of such a conditions is questioned, both because it is hard to find a pure case of Gerstmann's syndrome, and because the four symptoms do not have an obvious shared sub-function that can be affiliated with the angular gyrus (Rusconi, Pinel, Dehaene, & Kleinschmidt, 2010). However, the position against Gerstmann's syndrome can be considered a case of *argument from ignorance.* The lack of obvious sub-functions that underlie the four affected abilities, might be partially due to our lack of understanding of the neural mechanisms for these abilities. One potential explanation, at least to explain the relationship between finger sense and number processing, is that number processing uses finger schemas. Dehaene et al. (2003) originally dismissed this explanation, proposing that "… the syndrome may represent a happenstance conjunction of distinct, but dissociable, deficits that frequently co-occur due to a common vascularisation, and that are only loosely connected at the functional level due to the overarching spatial and sensorimotor functions of the parietal lobe" (p.493). According to this localizationist approach, there is no functional (casual) relation between finger and number representations, and the neuropsychological findings are due to large lesions affecting neural correlates of both finger and number representations, which are in close proximity in the parietal lobe.

Contrary to the localizationist approach, there is accumulating evidence for a functional relation between finger and number representations. In a study where excitability in hand muscles was measured during a visual parity judgment task, involving numbers between one and nine, modulation of right hand muscles, but not the left hand, was found with right-handed subjects (14 out of 16 started finger counting with their right hands). The effect was stronger for numbers between 1 and 4 (Brett et al., 2002). In another study, enumeration of dots on the screen both with numbers and letters were found to increase the corticospinal excitability of hand muscles (Sato et al., 2007). Because the effect was found both for enumeration with numbers and letters, the authors proposed that finger circuits are involved whenever a set of items have to be matched with the elements of an ordered series. In a dual-task study Michaux et al. (2013) showed that finger movements interfered more with arithmetic, compared to feet movements, and that the interference of finger movements on addition and subtraction was more than it was on multiplication, with adult participants. They proposed that the finger counting-based strategies used in childhood for addition and subtraction, but not for multiplication, leads to an early grounding of addition and subtraction processes in finger representations. Soylu and Newman (2016) followed up on these findings and studied the interaction between arithmetic difficulty and tapping complexity in a dual-task fMRI study. They reported differential interference of finger movements on single-digit addition, compared to double-digit addition, and traced this effect to the modulation of activity in the left angular gyrus. Additionally, Berteletti and Booth (2015) investigated activation of finger somatosensory and motor areas during single-digit subtraction and multiplication tasks, with children (8 to 13 years of age).

They found significant activations in motor areas only for subtraction and not multiplication, suggesting reliance on finger counting strategies. They also reported greater somatosensory activation for subtraction problems with larger operands, indicating greater reliance on finger representations. Together these studies provide further support for a functional relation between fingers and number processing, possibly due to a developmental association that can be traced back to early finger counting experiences.

Bridging Mathematics Education and Neurocognitive Research: A Roadmap for Future Research

The research reviewed so far shows that children's initial finger counting experiences, as well as different forms of both number related and non-number related hand and finger experiences, might be crucial in establishing the number processing network and in paving the way for learning of more advanced mathematics topics. To explore the implications of this body of research, we should shift our focus to which forms of finger-based activities can help support mathematics learning, and how finger-based indicators (e.g., counting, fine motor skills, finger sense) can help diagnose potential problems and predict future performance. Here we pose a set of research questions to provide a roadmap for future research.

The Relation between Finger Sense, Perceptual Units and Counting Types

Earlier in this paper we detailed how Steffe, Glasersfeld and colleagues (Glasersfeld, 1981; Steffe et al., 1988, 1983; Steffe & Glasersfeld, 1985) provided an account for how the fundamental perceptual ability to perceive discrete "things", by extracting aspects of the sensorimotor experience, lays out the foundation for understanding whole-part relations, and eventually for different forms of counting (i.e., motoric, figural, abstract). Even though there is accumulating evidence for a relation between finger sense and numerical ability, what the origin of this relation is, and how far we can trace it back in the developmental process is not clear. Steffe, Glasersfeld and colleagues' work point us in one direction. If extracting discrete unitary items from the fuzzy stream of sensorimotor experience, and applying these heuristics to construct an understanding of plurality and numerosity lead to development of number sense then early sensorimotor experiences with fingers might have a crucial role for development of numerical skills. Glasersfeld (1981) defined the initial step of perceiving "a thing" as extracting multimodal features from a background and integrating these features: "We do divide our visual, auditory, and tactual fields of experience into separate parts which, in our cognitive organization, then become individual items or "things."" (p.86). Each finger is a distinct sensorimotor item. One of the first, if not the first, set of discrete items infants encounter are their fingers. At birth the multimodal networks that allow coordination of sensory (tactile), motor, and visual modalities are not fully developed yet. These *reentrant networks* (Edelman, 1987; Smith & Gasser, 2005) develop during diverse multimodal bodily experiences. Piaget (1954) provided a vivid example of how this happens with an eight month old infant:

> "Laurent looks at his hands most attentively, as if he did not know them. He is alone in his bassinet, his hands motionless, but he constantly moves his fingers and examines them. After this he moves his hands slowly, looking at them with the same interested expression. Then he joins them and separates them more slowly while continuing to study the phenomenon; he ends by scratching his covers; striking them, etc., but watching his hands the whole time." (p.232)

If constructing an understanding of sensorimotor units, pluralities, and collections is a prerequisite for counting and number sense, and finger-based interactions are gateways to building these competencies, then hereditary and developmental differences in in the functioning of the finger sensorimotor system might constrain development of the prerequisite competencies for numerical development.

While there is accumulating research on how finger sense (Newman, 2016; Noel, 2005; Reeve & Humberstone, 2011) and fine motor skills (Luo, Jose, Huntsinger, & Pigott, 2007) correlate with and predict some numerical abilities, starting with kindergarten age, the ontogeny of the these relations is not clear. If the finger and number relation goes back to early and pre-verbal stages of development, then there is need for longitudinal studies tracing how early developments in the sensorimotor system set the stage for later numerical competencies.

What Can Finger Counting Habits Tell Us?

In several lab studies finger counting habits (the way one counts on her fingers from one to ten) showed significant effects in performance measures. Newman and Soylu (2014) reported higher addition performance for both adults and children (all right-handed) who were right starters (start counting with their right hands; numbers one to five are matched with fingers on the right hand, whereas numbers six to ten match with the left), compared to left-starters. Additionally, right-starter adults had significantly higher digit-span scores. The researchers explained the results in terms of hemispherical lateralization differences between right- and left-starters: while right-starters had a more left-dominant number processing network (which is the case for most right-handed individuals), left-starters were hypothesized to have a more bilateral number representation, which increases reliance on communication across the two hemispheres, thereby decreasing performance. In an earlier study, Tschentscher et al. (2012) provided supporting evidence for this explanation. In an fMRI study with adult participants, they found that in a categorization task (numbers vs. meaningless words or symbols), both number words and Arabic numerals (from one to nine) activated the finger motor network contralateral to the hand, matching with the finger counting configuration (e.g., right-starters activated the left motor areas, whereas left-starters activated right motor areas for numbers from one to five).

Domahs et al. (2010) compared the performance of deaf German signers, and non-deaf German and Chinese adults in a number comparison task using Arabic numerals. The German and Chinese groups differed in their finger counting habits in that while German participants (both impaired and hearing) used one hand for numbers smaller than six, and two hands for number larger than five, the Chinese participants used only one hand to sign all numbers smaller than and equal to 10. Both German groups (impaired and hearing) showed a decrease in performance in the comparison task, not predicted by the magnitude of the numbers alone, when at least one of the numbers required use of both hands for signing. This effect was not observed for Chinese participants. This study supports the idea that, independent of whether number signing is still actively used (which is the case for hearing-impaired German signers but not for the hearing German group), finger counting habits modulate performance.

In addition to lateralization effects, the chunking of numbers in groups of five (due to having five fingers on each hand) during finger counting seems to affect number processing. Domahs, Krinzinger and Willmes (2008) found above chance level split-five errors with first-grade children during addition and subtraction problems involving two double-digit numbers. Split-five errors are characterized by errors deviating by exactly ±5 from the correct result (e.g., 33 – 17 = 11). This result implies that children's mental representations for large numbers inherit the sub-base-five property of their hands.

What can we learn from finger counting habits for populations with mathematics learning disabilities and other cognitive disorders*?* So far studies on how finger counting habits interact with mathematics performance have mostly focused on children and adults without diagnosed disabilities. As we discussed earlier, children with mathematics learning difficulties have a harder time transitioning from finger counting to verbal counting and retrieval strategies (Bryant, 2009; Geary, 2004; Jordan & Hanich, 2000; Jordan et al., 2008), however, to date there are no studies investigating patterns of specific finger counting habits (e.g., right vs. left starter, symmetric vs. asymmetric configurations, anatomical vs. non-anatomical ordering) in children with mathematics learning disabilities, or other cognitive disorders that can impact mathematics learning and performance. If there are specific patterns, these can help diagnose and characterize different mathematics learning disabilities, and help design new interventions.

For example, patients diagnosed with autism (Gunter, Ghaziuddin, & Ellis, 2002), ADHD (Roessner, Banaschewski, Uebel, Becker, & Rothenberger, 2004), dyslexia (von Plessen et al., 2002), and developmental language disorder (Herbert et al., 2004) were found to have abnormalities in the size and function of their corpus callosum (the sets of nerve fibers connecting the left and right hemispheres; see van der Knaap & van der Ham, 2011 for a review) and lateralization patterns. It is possible that the abnormalities with corpus callosum, which affect interhemispheric communication, lead to differences in finger counting habits as well. If so, these can be of diagnostic value.

Development of Finger Skills in the Preschool Period

Findings from disparate studies show that, in addition to finger counting, fine motor skills (Luo et al., 2007), as well as finger sense (Noel, 2005; Reeve & Humberstone, 2011; Wasner et al., 2016) predict various mathematical competencies. We don't know enough about how these skills co-develop. We need longitudinal studies to investigate how finger sense, finger counting, fine motor abilities and mathematical skills co-develop and change between the ages of 4 and 6, when children transition from mere counting to arithmetic with finger counting, and eventually to fact retrieval and complex arithmetic calculations. Such studies can provide us with new insights on how the development of finger skills is linked to the development of numerical competency, and the developmental origins of mathematics learning disabilities. Insights gained can also shift our perspective about reasons for extended reliance on finger counting in children with mathematics disabilities. It is possible that the early developmental trajectory of finger skills signal future problems with mathematics development.

The early bodily experiences of children today differ significantly from previous generations due to availability of and early exposure to technology. How does early interaction with technology affect finger skills and what are the indirect effects on numerical development? If early bodily experiences are crucial for setting the stage for later numerical development, there are reasons to be concerned about how children's extensive interaction with technology can affect development of bodily – and in particular finger- skills, which then can have an effect on numerical development. In the U.S., preschool children are exposed to an average of 4.1 hours of screen time per day (Tandon, Zhou, Lozano, & Christakis, 2011). Screen time correlates with obesity, attentional and behavioral problems, and low school performance (Laurson et al., 2008; Swing, Gentile, Anderson, & Walsh, 2010). Apart from the more general effects of screen time, an unexplored area is how opportunities to develop manual dexterity and finger skills during play are being replaced with passive viewing (e.g., TV) and alternative modes of interaction with a wide range of devices (e.g., smartphones, game consoles). Given that clicking, scrolling and dragging can hardly match the richness of sensorimotor experiences that other forms of interaction with the physical world can afford, there is desperate need for research that can provide a critical evaluation of how interaction with a wide range of technologies affect finger sense and fine motor skills, and how differences in the development of these skills affect early numerical learning.

The research on the relation between fingers and numerical development can also guide development of new learning technologies. For example, *Touch Counts* is one such instructional program that harnesses affordances of new modes of interaction, touchscreen in this case, to provide learning experiences aligned with research on embodied cognition and the relevance of finger-based interactions for numerical development (Sinclair & Heyd-Metzuyanim, 2014). Touch Counts aims to support number sense by allowing enhanced engagement with number concepts through rich finger-based interactions. For example to improve counting skills learners can touch to the screen with multiple fingers to create numbered discs, one for each finger. As long as the learner's fingers stay on the screen the disc stays attached to it, when let go the discs fall to the bottom of the screen, due to the virtual gravity. Every time one or more fingers touch to the screen a matching number of discs are created (added on top of the existing ones), allowing representation of numbers more than 10. Touch Counts also targets associating multiple representations for numbers; finger-based, symbolic, non-symbolic, and auditory, by providing multi-modal interactions that involve these representations. Further interpretation of research findings for learning design efforts can lead to new instructional programs, software, and interventions to facilitate numerical development.

The Impact of the Home Environment

The home environment can be another factor that impacts the development of both finger and numerical skills. There has been extensive research regarding the impact of the home environment on reading (e.g., Niklas & Schneider, 2013) and mathematical skills (Anders et al., 2012; Melhuish et al., 2008). However, little is known about how home environment affects use of fingers during numerical development. Just as early exposure to literacy is important to the development of phonology and then reading skills, early exposure to numeracy is important to mathematical development, including the development of the neural pathways that support mathematical achievement. Part of this early exposure involves such experiences as observing a parent pointing to a quantity with finger gestures, or imitating finger counting by vocalizing number words, initially in arbitrary ways; until realizing that each vocalization matches with the movement of a single finger. The amount of exposure to experiences like these are likely to vary across different home environments and further research is needed to explore the impact of early finger-based numerical experiences on development of mathematical skills.

Finger-Based Interventions to Improve Mathematical Performance

In the reading literature there are several studies that examine the impact of phonological training on reading skill (Temple et al., 2003; Wagner, Torgesen, & Rashotte, 1994). For instance, Keller and Just (2009) examined a group of, non-dyslexic, poor readers who underwent 100 hours of remediation focused on word-level decoding skills with a strong phonological skill focus. They found significant improvements pre- and post-training in word recognition skills. Additionally, the brain structure itself was altered as a result of training. Post-training, white matter tracts (denoting connectivity across primary regions for reading) in poor readers changed to look more similar to that of good readers. The many phonological training studies show that training on a component process that is fundamental to reading can improve reading performance. A similar approach can be applied to the relation between finger skills and numerical performance. Training on finger processing (i.e., finger sense, fine motor ability, finger counting) might improve numerical and mathematical competency in a similar way to how training on phonology improves reading performance.

There are a few studies examining the impact of finger training on arithmetic performance. Gracia-Bafalluy and Noel (2008) performed a finger training study with first graders (6-7 year olds) and found improvements both in finger sense and measures of numerical competence after training. However, these results should be considered with caution. Fischer (2010) pointed out that the results Gracia-Bafalluy and Noel (2008) reported might be, at least partially, due to simple regression toward the mean. Additionally, Zafranas (2004) reported improvements in hand movements, spatial memory, and arithmetic performance in a study of piano keyboard training. While it is not clear whether the music training or the keyboard/finger training led to better arithmetic performance, the results are consistent with the hypothesis that finger training is linked to arithmetic performance. It is possible that among other factors (Anvari, Trainor, Woodside, & Levy, 2002; Vaughn, 2000), development of finger skills (both tactile and motor) mediate the benefits observed from music training on mathematical skills. No research so far explicitly focused on how music training affects mathematics development and performance through development of finger skills, and this is a topic ripe for research, with important implications for music and mathematics curricula at schools.

Conclusion

While number processing is often conceived as abstract, the systems that support number processing are related to systems that allow us to engage in seemingly mundane bodily tasks. An evolutionary perspective can help us understand why and how mathematical abilities are grounded in bodily systems. Mathematics being a relatively recent cultural invention, there was not enough in our evolutionary history to develop brain systems that are explicitly dedicated to mathematical cognition. Like other recent steps in human evolution, such as verbal abilities and writing, mathematical abilities had to rely on and reuse existing systems and abilities. Evolution of hands plays a special role in human evolution. We use (and have used in the past) our hands to build tools, to manipulate our environment, and to communicate. Hands are represented disproportionately in both somatosensory and primary motor areas of the brain. The sensorimotor system that supports functioning of the hands mainly involves the sensory (tactile), motor, and visual modalities. These three modalities coordinate to allow for visually guided movement of the hands to engage in a wide range of tasks, from manipulating objects (Maravita & Iriki, 2004) to gesture based communication (Arbib, 2005). In addition, our tactile experiences, which also involve motor and visual modalities, provide rich information (e.g., temperature, texture, hardness) about different characteristics of the objects and other living things around us.

Neurocognitive studies provide insights about how the finger sensorimotor systems supports and scaffolds number processing; how behavioral indicators for fingers (e.g., finger sense, finger counting habits) correlate with and predict numerical indicators (e.g., subitizing, counting, arithmetic), and how neural correlates of finger (e.g., finger sense, finger motor movements) and number (e.g., magnitude processing, arithmetic) skills interact and overlap. Most neurocognitive studies on the relevance of fingers for number skills are theoretically framed by embodied approaches to cognition, and consider involvement of the finger sensorimotor system in number processing as an aspect of the embodiment of mathematics (Moeller et al., 2012; Wasner, Moeller, Fischer, & Nuerk, 2015).

The mathematics education research presented here share a constructivist orientation, according to which number concepts are constructed through bodily interactions with the world. According to this approach, early interactions with the world set the foundation for understanding number concepts, and through development, number skills progressively become more independent of the physical interactions and representations used early on; eventually to be replaced by more abstract and formal constructs. This body of research gives us a sense of how pre-verbal abilities that lay the foundation for later number concepts develop through ordinary bodily interactions. It also sheds light on how number concepts are constructed from a first-person perspective, by capturing children's ways of thinking through clinical interviews.

The two bodies of research differ in terms of methodologies and levels of analysis, as well as the theoretical framing of why and how bodily interactions, in particular with fingers, are relevant (if not central) to numerical development. Constructivist research, exemplified by the works of Glasersfeld, Steffe and colleagues (Glasersfeld, 1981; Steffe et al., 1988, 1983; Steffe & Glasersfeld, 1985), showed how number concepts are constructed as a result of bodily interactions with the world, and how these concepts go through representational transitions through development. Interactions with our hands and fingers set the foundation for concept formation, understanding of one-to-one correspondence, and quantity. Fingers also act as figural representations for quantity, and are used both for number gestures, and for finger counting. Mathematics education also provides us with accounts of how finger counting strategies for arithmetic evolve over time, eventually to be replaced by more memory- and rule-based mental calculation strategies. For the most part, this body of research does not explain how the brain systems for number and finger processing change through development, and how they relate to each other. It also does not focus extensively on how fingers relate to number processing in adulthood, likely because the relation between fingers (and the body in general) and mathematics is assumed to become less relevant in adulthood, due to the development of more abstract representations and calculation strategies.

Neurocognitive research, on the other hand, focuses on the brain systems that support both finger and number processing (e.g., Andres, Seron, & Olivier, 2007; Soylu & Newman, 2016; Tschentscher et al., 2012), and how a wide range of finger-based indicators (finger sense, finger tapping, finger counting habits) relate to and predict performance in number tasks, for example subitizing, counting, magnitude processing, arithmetic (e.g., Crollen & Noel, 2015; Newman, 2016; Noel, 2005). It also presents us with evidence for embodiment of mathematics, and relevance of finger-based of indicators in adult populations.

Grouping a wide range of work on the relation between fingers and mathematics under two main categories; constructivist mathematics education and embodied neurocognitive research, reduces the complexity within each body of research. Nevertheless, this categorization provides us with ways of comparing the two bodies of research, and looking at how the two can be bridged in synergistic ways to provide a more complete picture of how our body scaffolds and grounds number skills, how this grounding takes place across different levels of analysis (e.g., first-person, behavioral, neural), and how the finger-based measures can help with understanding developmental patterns and skills in mathematics.

Instead of characterizing fingers as physical manipulatives that are relevant to numerical cognition during only a limited window of development, we argue that features of numerical cognition may be grounded in the finger sensorimotor system. This grounding may be similar to the relationship between phonological awareness and reading. Phonological awareness (awareness of the speech sounds in the language and how they map onto letters or syllables) has been shown to be a predictor of success in learning to read (Treiman & Zukowski, 1991) and deficits in phonological awareness are typical in dyslexia (Temple et al., 2003). Additionally, while congenitally hearing-impaired individuals (who are unaware of the speech sounds) do learn to read, it is often laborious and many have very poor reading skills (Holt, 1994). Analogously, early interactions with the world may help the emergence of networks in the brain linking visuospatial, tactile and motor modalities for hands and fingers and, later, figural use of fingers to represent and process quantitates through number gestures. In short, finger skills (e.g., finger sense, finger counting) may be a doorway to mathematical competence in the same way that phonology is for reading. The dependency of numerical development on finger skills, and the sustained involvement of the finger sensorimotor system in mathematical cognition, extending into adulthood, paints a different picture for the finger and mathematics relation than is often assumed. While constructivist literature in mathematics education provides accounts for how early bodily interactions support construction of foundational number concepts (e.g., 1-to-1 correspondence, composite unit understanding), and how finger counting strategies support development of arithmetic skills, neurocognitive research provides insights about the neural mechanisms that underlie the finger and mathematics relation, and how finger-based measures (e.g., finger sense, finger counting habits) predict and interact with number skills throughout the development and adulthood. Taken together, these two bodies of literature paint a more comprehensive picture of the finger and mathematics relation, and the embodiment of mathematics in general. Bridging the neurocognitive research and mathematics education gap opens the way for previously unexplored areas of research (e.g., effects of finger training, effects of home environment, mathematics learning disabilities), and can help with the design of new interventions and evidence-based learning technologies for early mathematics learning.

References

Alibali, M. W., & DiRusso, A. A. (1999). The function of gesture in learning to count: More than keeping track. *Cognitive Development*, *14*(1), 37–56.

Anders, Y., Rossbach, H. G., Weinert, S., Ebert, S., Kuger, S., Lehrl, S., & von Maurice, J. (2012). Home and preschool learning environments and their relations to the development of early numeracy skills. *Early Childhood Research Quarterly*, *27*(2), 231–244. http://doi.org/10.1016/j.ecresq.2011.08.003

Anderson, M. L. (2010). Neural reuse: a fundamental organizational principle of the brain. *The Behavioral and Brain Sciences*, *33*(4), 245-66-313. http://doi.org/10.1017/S0140525X10000853

Andres, M., Davare, M., Pesenti, M., Olivier, E., & Seron, X. (2004). Number magnitude and grip aperture interaction. *Neuroreport*, *15*, 2773–2777. http://doi.org/00001756-200412220-00017

Andres, M., Seron, X., & Olivier, E. (2007). Contribution of hand motor circuits to counting. *Journal of Cognitive Neuroscience*, *19*(4), 563–576.

Anvari, S., Trainor, L., Woodside, J., & Levy, B. (2002). Relations among musical skills, phonological processing, and early reading ability in preschool children. *Journal of Experimental Child Psychology*, *83*, 111–130.

Arbib, M. (2005). From monkey-like action recognition to human language: An evolutionary framework for neurolinguistics. *Behavioral and Brain Sciences*, *28*(2), 105–124.

Badets, A., Andres, M., Di Luca, S., & Pesenti, M. (2007). Number magnitude potentiates action judgements. *Experimental Brain Research*, *180*(3), 525–534.

Baroody, A. J. (1987). The development of counting strategies for single-digit addition. *Journal* *for Research in Mathematics Educatioin*, *18*(2), 141–157.

Barrett, T. M., Traupman, E., & Needham, A. (2008). Infants' visual anticipation of object structure in grasp planning. *Infant Behavior and Development*, *31*(1), 1–9. http://doi.org/10.1016/j.infbeh.2007.05.004

Bender, A., & Beller, S. (2012). Nature and culture of finger counting: Diversity and representational effects of an embodied cognitive tool. *Cognition*, *124*(2), 156–182. http://doi.org/10.1016/j.cognition.2012.05.005

Benton, A. (1955). Development of finger-localization capacity in school children. *Child Development*, *26*(4), 225–230. http://doi.org/10.2307/1126283

Benton, A. (1979). The neuropsychological significance of finger recognition. In M. Bortner (Ed.), *Cognitive Growth and Development*. New York: Brunner-Mazel.

Berteletti, I., & Booth, J. R. (2015). Perceiving fingers in single-digit arithmetic problems. *Frontiers in Psychology*, *6*(MAR), 1–10. http://doi.org/10.3389/fpsyg.2015.00226

Berteletti, I., & Booth, J. R. (2016). Finger representation and finger-based strategies in the acquisition of number meaning and arithmetic. In D. Berch, D. Geary, & K. Koepke (Eds.), *Development of Mathematical Cognition* (pp. 109–139). Elsevier. http://doi.org/10.1016/B978-0-12-801871-2.00005-8

Boaler, J., & Chen, L. (2016). Why Kids Should Use Their Fingers in Math Class. *The Atlantic*.

Bryant, D. P. (2009). Commentary on early identification and intervention for students with mathematics difficulties. *Journal of Learning Disabilities*, *38*, 340–345. http://doi.org/10.1177/00222194050380041001

Butterworth, B. (1999). *What counts: How every brain is hardwired for math*. The Free Press.

Butterworth, B. (2005). The development of arithmetical abilities. *Journal of Child Psychology* *and Psychiatry*, *46*(1), 3–18. http://doi.org/10.1111/j.1469-7610.2004.00374.x

Clark, A. (1999). An embodied cognitive science? *Trends In Cognitive Sciences*, *3*(9), 345–351.

Cowan, R. (2013). Does it all add up? Changes in children's knowledge of addition, combinations, strategies, and principles. In A. J. Baroody & A. Dowker (Eds.), *The development of arithmetic concepts and skills: Constructive adaptive expertise* (pp. 35–74). Routledge.

Critchleey, M. (1953). Parietal lobes. *Giornale Di Psichiatria E Di Neuropatologia*, *81*(4), 872–873.

Crollen, V., & Noel, M.-P. (2015). The role of fingers in the development of counting and arithmetic skills. *Acta Psychologica*, *156*, 37–44.

de Freitas, E., & Sinclair, N. (2013). New materialist ontologies in mathematics education: the body in/of mathematics. *Educational Studies in Mathematics*, *83*(3), 453–470.

Dehaene, S., & Cohen, L. (1995). Towards an anatomical and functional model of number processing. *Mathematical Cognition*, *1*(1), 83–120.

Dehaene, S., Piazza, M., Pinel, P., & Cohen, L. (2003). Three parietal circuits for number processing. *Cognitive Neuropsychology*, *20*(3), 487–506. http://doi.org/10.1080/02643290244000239

Devine, A. (2013). Developmental dyscalculia. *Pediatric Neurology*, *24*(5), 337–342.

DeWind, N. K., & Brannon, E. M. (2012). Malleability of the approximate number system: effects of feedback and training. *Frontiers in Human Neuroscience*, *6*(April), 1–10. http://doi.org/10.3389/fnhum.2012.00068

Di Luca, S., & Pesenti, M. (2008). Masked priming effect with canonical finger numeral configurations. *Experimental Brain Research*, *185*(1), 27–39.

Domahs, F., Krinzinger, H., & Willmes, K. (2008). Mind the gap between both hands: evidence for internal finger-based number representations in children's mental calculation. *Cortex*, *44*, 359–67. http://doi.org/10.1016/j.cortex.2007.08.001

Domahs, F., Moeller, K., Huber, S., Willmes, K., & Nuerk, H.-C. C. (2010). Embodied numerosity: Implicit hand-based representations influence symbolic number processing across cultures. *Cognition*, *116*(2), 251–266. http://doi.org/10.1016/j.cognition.2010.05.007

Edelman, G. M. (1987). *Neural Darwinism: The theory of neuronal group selection*. New York: Basic Books.

Fayol, M., Barrouillet, P., & Marinthe, C. (1998). Predicting arithmetical achievement from neuro-psychological performance: A longitudinal study. *Cognition*, *68*(2), B63–B70.

Fayol, M., & Seron, X. (2005). About numerical representations: Insights from neuropsychological, experimental, and developmental studies. In J. Campbell (Ed.), *Handbook of Mathematical cognition* (pp. 3–22). New York: Psychology Press.

Fischer, J.-P. (2010). Numerical performance increased by finger training: a fallacy due to regression toward the mean? *Cortex*, *46*, 272–3. http://doi.org/10.1016/j.cortex.2008.06.010

Fuhs, M. W., & Mcneil, N. M. (2013). ANS acuity and mathematics ability in preschoolers from low-income homes: Contributions of inhibitory control. *Developmental Science*, *16*, 136–148. http://doi.org/10.1111/desc.12013

Fuson, K. (1982). An analysis of the counting-on solution procedure in addition. In T. Carpenter (Ed.), *Addition and subtraction: A cognitive perspective* (pp. 67–81). Erlbaum.

Fuson, K. (1988). *Children's counting and concepts of number*. *Cognitive Development* (Vol. 31). Springer-Verlag Publishing. http://doi.org/10.1017/s0140525x08005724

Gallese, V., & Lakoff, G. (2005). The brain's concepts: the role of the sensory-motor system in conceptual knowledge. *Cognitive Neuropsychology*, *22*, 455–79. http://doi.org/10.1080/02643290442000310

Geary, D. C. (1990). A componential analysis of an early learning deficit in mathematics. *Journal of Experimental Child Psychology*, *49*, 363–383. http://doi.org/10.1016/0022-0965(90)90065-G

Geary, D. C. (2004). Mathematics and Learning Disabilities. *Journal of Learning Disabilities*, *37*, 4–15. http://doi.org/10.1177/00222194040370010201

Geary, D. C., & Wiley, J. G. (1991). Cognitive addition: Strategy choice and speed-of-processing differences in young and elderly adults. *Psychology and Aging*, *6*(3), 474–83.

Gersten, R., & Chard, D. (1999). Rethinking arithmetic instructions for students with mathematical disabilities. *The Journal of Special Education*, *33*(1), 18–28.

Gerstmann, J. (1940). Syndrome of finger agnosia, disorientation for right and left, agraphia and acalculia. *Archives of Neurology and Psychiatry*, *44*, 398–408.

Glasersfeld, E. von. (1981). An attentional model for the conceptual construction of units and number. *Journal for Research in Mathematics Education*, *12*(2), 83–94.

Gracia-Bafalluy, M., & Noel, M. P. (2008). Does finger training increase young children's numerical performance? *Cortex*, *44*, 368–375.

Greenough, W. T., Black, J. E., & Wallace, C. S. (1987). Experience and brain development. *Child Development*, *58*, 539–559. http://doi.org/10.2307/1130197

Gunter, H. L., Ghaziuddin, M., & Ellis, H. D. (2002). Asperger syndrome: Tests of right hemisphere functioning and interhemispheric communication. *Journal of Autism and Developmental Disorders*, *32*, 263–281. http://doi.org/10.1023/A:1016326701439

Halberda, J., & Feigenson, L. (2008). Developmental change in the acuity of the "Number

Sense”: The Approximate Number System in 3-, 4-, 5-, and 6-year-olds and adults. *Developmental Psychology*, *44*, 1457–1465. http://doi.org/10.1037/a0012682

Hanich, L. B., Jordan, N. C., Kaplan, D., & Dick, J. (2001). Performance across different areas of mathematical cognition in children with learning difficulties. *Journal of Educational Psychology*, *93*, 615–626. http://doi.org/10.1037/0022-0663.93.3.615

Herbert, M. R., Ziegler, D. A., Makris, N., Filipek, P. A., Kemper, T. L., Normandin, J. J., … Caviness, V. S. (2004). Localization of white matter volume increase in autism and developmental language disorder. *Annals of Neurology*, *55*, 530–540. http://doi.org/10.1002/ana.20032

Holt, J. (1994). Classroom attributes and achievement test scores for deaf and hard of hearing students. *American Annals of the Deaf*, *139*, 430–437. http://doi.org/10.1353/aad.2012.0274

Jordan, N. C., & Hanich, L. B. (2000). Mathematical thinking in second-grade children with different forms of LD. *Journal of Learning Disabilities*, *33*, 567–578. http://doi.org/10.1177/002221940003300605

Jordan, N. C., Hanich, L. B., & Kaplan, D. (2003). A longitudinal study of mathematical competencies in children with specific mathematics difficulties versus children with comorbid mathematics and reading difficulties. *Child Development*, *74*, 834–850. http://doi.org/10.2964/jsik.kuni0223

Jordan, N. C., Huttenlocher, J., & Levine, S. C. (1992). Differential calculation abilities in young children from middle- and low-income families. *Developmental Psychology*, *28*, 644–653. http://doi.org/10.1037/0012-1649.28.4.644

Jordan, N. C., Kaplan, D., Ola, L. N., & Locuniak, M. N. (2006). Number sense growth in kindergarten : A longitudinal investigation of children at risk for mathematics difficulties, *77*(1), 153–175.

Jordan, N. C., Kaplan, D., Ramineni, C., & Locuniak, M. N. (2008). Development of number combination skill in the early school years: When do fingers help? *Developmental Science*, *11*, 662–668. http://doi.org/10.1111/j.1467-7687.2008.00715.x

Jordan, N. C., Levine, S. C., & Huttenlocher, J. (1994). Development of calculation abilities in middle- and low-income children after formal instruction in school. *Journal of Applied Developmental Psychology*, *15*, 223–240. http://doi.org/10.1016/0193-3973(94)90014-0

Jordan, N. C., & Montani, T. O. (2015). Cognitive arithmetic and problem solving: a comparison of children with specific and general mathematics difficulties. *Journal of Learning Disabilities*, *30*, 624–634. http://doi.org/10.1177/002221949703000606

Keller, T. A., & Just, M. A. (2009). Altering cortical connectivity: remediation-induced changes in the white matter of poor readers. *Neuron*, *64*(5), 624–31. http://doi.org/10.1016/j.neuron.2009.10.018

Klein, E., Suchan, J., Moeller, K., Karnath, H.-O., Knops, A., Wood, G., … Willmes, K. (2014). Considering structural connectivity in the triple code model of numerical cognition: differential connectivity for magnitude processing and arithmetic facts. *Brain Structure & Function*. http://doi.org/10.1007/s00429-014-0951-1

Lakoff, G., & Nunez, R. E. (2000). *Where mathematics comes from: How the embodied mind brings mathematics into being*. Basic Books.

Laurson, K. R., Eisenmann, J. C., Welk, G. J., Wickel, E. E., Gentile, D. A., & Walsh, D. A. (2008). Combined influence of physical activity and screen time recommendations on childhood overweight. *Journal of Pediatrics*, *153*, 209–214. http://doi.org/10.1016/j.jpeds.2008.02.042

Libertus, M. E., Feigenson, L., & Halberda, J. (2011). Preschool acuity of the approximate number system correlates with school math ability. *Developmental Science*, *14*, 1292–1300. http://doi.org/10.1111/j.1467-7687.2011.01080.x

Long, I., Malone, S. A., Tolan, A., Burgoyne, K., Heron-Delaney, M., Witteveen, K., & Hulme, C. (2016). The cognitive foundations of early arithmetic skills: It is counting and number judgment, but not finger gnosis, that count. *Journal of Experimental Child Psychology*, *152*, 327–334. http://doi.org/10.1016/j.jecp.2016.08.005

Luo, Z., Jose, P. E., Huntsinger, C. S., & Pigott, T. D. (2007). Fine motor skills and mathematics achievement in East Asian American and European American kindergartners and first graders. *British Journal of Developmental Psychology*, *25*, 595–614. http://doi.org/10.1348/026151007X185329

Maravita, A., & Iriki, A. (2004). Tools for the body (schema). *Trends in Cognitive Sciences*, *8*(2), 79–86. http://doi.org/10.1016/j.tics.2003.12.008

Mayer, E., Martory, M. D., Pegna, A. J., Landis, T., Delavelle, J., & Annoni, J. M. (1999). A pure case of Gerstmann syndrome with a subangular lesion. *Brain*, *122*(6), 1107.

Mazzocco, M. M., Feigenson, L., & Halberda, J. (2011). Preschoolers' precision of the approximate number system predicts later school mathematics performance. *PLoS ONE*, *6*(9), 1–8. http://doi.org/10.1371/journal.pone.0023749

Mazzocco, M. M., & Thompson, R. E. (2005). Kindergarten predictors of math learning disability. *Learning Disabilities Research & Practice*, *20*(3), 142–155. http://doi.org/10.1111/j.1540-5826.2005.00129.x

Melhuish, E., Sylva, K., Sammons, P., Siraj-Blatchford, I., Taggart, B., & Phan, M. (2008). Effects of the Home Learning Environment and preschool center experience upon literacy and numeracy development in early primary school. *Journal of Social Issues*, *64*(1), 95–114. http://doi.org/10.1111/j.1540-4560.2008.00550.x

Michaux, N., Masson, N., Pesenti, M., & Andres, M. (2013). Selective interference of finger movements on basic addition and subtraction problem solving. *Experimental Psychology*, *60*(3), 197–205. http://doi.org/10.1027/1618-3169/a000188

Moeller, K., Fischer, U., Link, T., Wasner, M., Huber, S., Cress, U., & Nuerk, H. (2012). Learning and development of embodied numerosity. *Cognitive Processing*, *13*, 271–4. http://doi.org/10.1007/s10339-012-0457-9

Moeller, K., Martignon, L., Wessolowski, S., Engel, J., & Nuerk, H. C. (2011). Effects of finger counting on numerical development the opposing views of neurocognition and mathematics education. *Frontiers in Psychology*, *2*(November), 1–5. http://doi.org/10.3389/fpsyg.2011.00328

Newman, S. (2016). Does finger sense predict addition performance? *Cognitive Processing*, *17*, 139–46. http://doi.org/10.1007/s10339-016-0756-7

Newman, S., & Soylu, F. (2014). The impact of finger counting habits on arithmetic in adults and children. *Psychological Research*, *78*(4), 549–56. http://doi.org/10.1007/s00426-013-0505-9

Niklas, F., & Schneider, W. (2013). Home literacy environment and the beginning of reading and spelling. *Contemporary Educational Psychology*, *38*(1), 40–50. http://doi.org/10.1016/j.cedpsych.2012.10.001

Noel, M. P. (2005). Finger gnosia: a predictor of numerical abilities in children? *Child Neuropsychology*, *11*, 413–430.

Nunez, R. (2012). On the Science of Embodied Cognition in the 2010s: Research Questions, Appropriate Reductionism, and Testable Explanations. *Journal of Learning Sciences*, *21*, 324–336. http://doi.org/10.1080/10508406.2011.614325

O'Brien, J., Spencer, J., Atkinson, J., Braddick, O., & Wattam-Bell, J. (2002). Form and motion coherence processing in dyspraxia: evidence of a global spatial processing deficit. *Neuroreport*, *13*, 1399–1402. http://doi.org/10.1097/00001756-200208070-00010

Penner-Wilger, M., & Anderson, M. L. (2013). The relation between finger gnosis and mathematical ability: why redeployment of neural circuits best explains the finding. *Frontiers in Psychology*, *4*(December), 877. http://doi.org/10.3389/fpsyg.2013.00877

Piaget, J. (1954). *The Child's Construction of Reality*. Routledge. http://doi.org/10.1037/11168-000

Piaget, J. (1970). *Genetic Epistemology*. New York: W. W. Norton.

Posner, M., & Rothbart, M. (2007). *Educating the human brain*. American Psychological Association. http://doi.org/doi:10.1037/11519-009

Ramani, G. B., & Siegler, R. (2008). Promoting broad and stable improvements in low-income children's numerical knowledge through playing number board games. *Child Development*, *79*, 375–94. http://doi.org/10.1111/j.1467-8624.2007.01131.x

Reeve, R., & Humberstone, J. (2011). Five-to 7-year-olds' finger gnosia and calculation abilities. *Frontiers in Psychology*, *2*.

Richardson, L. J. (1916). Digital reckoning among the ancients. *American Mathematical Monthly*, *23*(1), 7–13.

Roessner, V., Banaschewski, T., Uebel, H., Becker, A., & Rothenberger, A. (2004). Neuronal network models of ADHD - Lateralization with respect to interhemispheric connectivity reconsidered. *European Child and Adolescent Psychiatry*, *13*(1), 71–79.

http://doi.org/10.1007/s00787-004-1007-5

Rousselle, L., & Noel, M.-P. (2007). Basic numerical skills in children with mathematics learning disabilities: a comparison of symbolic vs non-symbolic number magnitude processing. *Cognition*, *102*(3), 361–395. http://doi.org/10.1016/j.cognition.2006.01.005

Roux, F. E., Boetto, S., Sacko, O., Chollet, F., & Tremoulet, M. (2003). Writing, calculating, and finger recognition in the region of the angular gyrus: a cortical stimulation study of Gerstmann syndrome. *Journal of Neurosurgery*, *99*(4), 716–727.

Rubinsten, O., & Henik, A. (2009). Developmental Dyscalculia: heterogeneity might not mean different mechanisms. *Trends in Cognitive Sciences*, *13*(2), 92–99. http://doi.org/10.1016/j.tics.2008.11.002

Rusconi, E., Walsh, V., & Butterworth, B. (2005). Dexterity with numbers: rTMS over left angular gyrus disrupts finger gnosis and number processing. *Neuropsychologia*, *43*, 1609–1624.

Saxe, G. B. (1981). Body parts as numerals: A developmental analysis of numeration among the Oksapmin in Papua New Guinea. *Child Development*, *52*, 306–316.

Schlaggar, B. L., & McCandliss, B. D. (2007). Development of neural systems for reading. *Annual Review of Neuroscience*, *30*, 475–503. http://doi.org/10.1146/annurev.neuro.28.061604.135645

Sfard, A. (1994). Reification as the Birth of Metaphor. *For the Learning of Mathematics*, *14*(1), 44–55.

Sinclair, N., & Heyd-Metzuyanim, E. (2014). Learning number with TouchCounts: The role of emotions and the body in mathematical communication. *Technology, Knowledge and Learning*, *19*(1–2), 81–99. http://doi.org/10.1007/s10758-014-9212-x

Smith, L., & Gasser, M. (2005). The development of embodied cognition: Six lessons from babies. *Artificial Life*, *11*(1–2), 13–29. http://doi.org/10.1162/1064546053278973

Soylu, F. (2011). Mathematical cognition as embodied simulation. In *Proceedings of the 33rd Annual Conference of the Cognitive Science Society*. Austin, TX: Cognitive Science Society.

Soylu, F. (2016). An embodied approach to understanding: Making sense of the world through simulated bodily activity. *Frontiers in Psychology*, *7*(December), 1–10. http://doi.org/10.3389/fpsyg.2016.01914

Soylu, F., & Newman, S. (2016). Anatomically ordered tapping interferes more with one-digit addition than two-digit addition: a dual-task fMRI study. *Cognitive Processing*, *17*(1), 67–77. http://doi.org/10.1007/s10339-015-0737-2

Soylu, F., Raymond, D., Gutierrez, A., & Newman, S. (2017). *The differential relationship between finger sense, and addition and subtraction: an fMRI study.* Manuscript submitted for publication.

Steffe, L., Cobb, P., & Glasersfeld, E. von. (1988). *Construction of arithmetical meanings and strategies*. *Recent research in psychology*. Springer. http://doi.org/10.1007/978-1-4612-3844-7

Steffe, L., & Glasersfeld, E. von. (1985). Helping children to conceive of number. *Recherches En Didactique Des Mathématiques*, *6*, 269–303.

Steffe, L., Glasersfeld, E. von, Richards, J., & Cobb, P. (1983). *Children's counting types: Theory, philosophy, and application*. New York: Praegar Scientific.

Svensson, H. (2007). *Embodied simulation as off-line representation*. *Science*. Linköping University.

Swing, E. L., Gentile, D. A., Anderson, C. A., & Walsh, D. A. (2010). Television and video game exposure and the development of attention problems. *Pediatrics*, *126*, 214–21. http://doi.org/10.1542/peds.2009-1508

Tandon, P. S., Zhou, C., Lozano, P., & Christakis, D. A. (2011). Preschoolers' total daily screen time at home and by type of child care. *Journal of Pediatrics*, *158*, 297–300. http://doi.org/10.1016/j.jpeds.2010.08.005

Temple, E., Deutsch, G. K., Poldrack, R. A., Miller, S. L., Tallal, P., Merzenich, M. M., & Gabrieli, J. D. E. (2003). Neural deficits in children with dyslexia ameliorated by behavioral remediation: evidence from functional MRI. *Proceedings of the National Academy of Sciences of the United States of America*, *100*, 2860–2865. http://doi.org/10.1073/pnas.0030098100

Treiman, R., & Zukowski, A. (1991). Levels of phonological awareness. In S. S. Brady & D. P. Shankweiler (Eds.), *Phonological Processes in Literacy* (pp. 67–83).

Tschentscher, N., Hauk, O., Fischer, M., & Pulvermuller, F. (2012). You can count on the motor cortex: Finger counting habits modulate motor cortex activation evoked by numbers. *Neuroimage*, *59*, 3139–3148.

van der Knaap, L. J., & van der Ham, I. J. M. (2011). How does the corpus callosum mediate interhemispheric transfer? A review. *Behavioural Brain Research*, *223*(1), 211–221. http://doi.org/10.1016/j.bbr.2011.04.018

Vaughn, K. (2000). Music and mathematics: Modest support for the oft-claimed relationship. *Journal of Aesthetic Education*, *34*(3/4), 149. http://doi.org/10.2307/3333641

von Plessen, K., Lundervold, A., Duta, N., Heiervang, E., Klauschen, F., Smievoll, A. I., … Hugdahl, K. (2002). Less developed corpus callosum in dyslexic subjects--a structural MRI study. *Neuropsychologia*, *40*, 1035–1044. http://doi.org/10.1016/S0028-3932(01)00143-9

Wagner, R. K., Torgesen, J. K., & Rashotte, C. a. (1994). Development of reading-related phonological processing abilities: New evidence of bidirectional causality from a latent variable longitudinal study. *Developmental Psychology*, *30*, 73–87. http://doi.org/10.1037/0012-1649.30.1.73

Wasner, M., Moeller, K., Fischer, M. H., & Nuerk, H.-C. (2015). Related but not the same: Ordinality, cardinality and 1-to-1 correspondence in finger-based numerical representations. *Journal of Cognitive Psychology*, *27*(4), 426–441. http://doi.org/10.1080/20445911.2014.964719

Wasner, M., Nuerk, H.-C., Martignon, L., Roesch, S., & Moeller, K. (2016). Finger gnosis predicts a unique but small part of variance in initial arithmetic performance. *Journal of Experimental Child Psychology*, *146*, 1–16. http://doi.org/10.1016/j.jecp.2016.01.006

Zafranas, N. (2004). Piano keyboard training and the spatial–temporal development of young children attending kindergarten classes in Greece. *Early Child Development and Care*, *174*(2), 199–211.